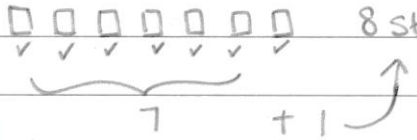
Specialist Mathematics Unit 1: Chapter 1

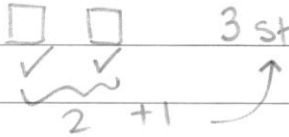
Ex 1A.

1.



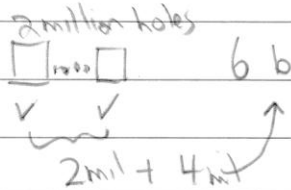
at least 1 question will be done by two or more students

2.



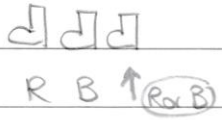
at least 1 class will contain two or more of the triplets

3.



at least 1 marker is possessed by each person

4.



he will pull out at least 2 of the same colour

5.

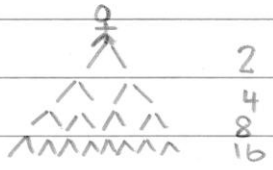
average of 100000 hairs

20 million people

no one has more than 1 million hairs on their head

there has to be more than 2 people with the same N° of hairs on their head

6.



- Siblings will have same ancestors ∴ not necessarily greater population
- Family clans will have overlapping relatives ∴ not neccesarily greater population in past.

7. 15 people in party, 14 others to shake hands with

a) max n° of handshakes = 14
shake hands with everyone else

b) min n° of handshakes = 0
don't shake hands with anyone

c) could some one have shaken hands will all 14 ∈ (AND) some one else with no one!
Not possible because the person that shook 14 hands would have included the one who didn't want to shake hands

8. If a polygon has 3 sides (p), then it is a triangle (q)

True $p \Leftrightarrow q$ *polygon is closed shape.

9. If Jenny is talking (p) her mouth is open (q)

$p \not\Leftrightarrow q$

(she could be yawning, eating etc.)

10. animal is platypus (p), then its a mammal (q)

platypus → mammal yes

mammal ↛ platypus NO!

$p \not\Leftrightarrow q$

(could be koala, whale etc.)

11. car out of petrol (p) it wont start (q)

$p \not\Leftrightarrow q$

could have dead battery etc..

12. points lie on the same straight line (p) then they are collinear (q)

$p \Leftrightarrow q$

13. If tomorrow is not Friday then today is not Thursday.

14. If a number is not even then it is not a multiple of two.

15. If a triangle is not scalene it doesn't have three different lengths.

16. If my lawn is not wet then my sprinklers are not on.

17. If Arm and gets up before Sam then it is a school day.

18. If a polygon is a Δ then its angles add to 180

a) $p \Leftrightarrow q$ true

b) If a polygon is not a Δ then its angles don't add to 180°

c) inverse is also true

19. If a positive integer has exactly 2 factors then its a prime N°

a) true

b) If a positive intege doesn't have exactly two factors then it is not a prime N°

c) inverse is also true

20. If the car battery is flat then the car won't start

a) true

b) If the car battery is not flat then the car will start

c) inverse is false (could be out of petrol)

21. If there are letters in my post box then the mailman has been

a) true.

b) If there are no letters in my post box, the mail man has not been

c) Inverse is false, you might not have any mail today.

22. If a number is even then it a multiple of 4.

a) False (eg 18 is not x by 4)

b) If a number is not even then it's not a multiple of 4

c) inverse is true, as odds are not multiples of 4

23. If a polygon has 5 sides then it's a pentagon

converse

• If a polygon is a pentagon then it has 5 sides

• inverse

If a polygon is not 5 sided

24. if a quad is a square the four angles are 90°

• converse

If the four angles are 90° then the quadrilateral is a square F. (rectangle!)

• Inverse. If a quadrilateral is not a square then the four angles are not 90° (False!) (could rectangle)

• contrapositive. If the four angles of a quadrilateral are not 90°, then the quadrilateral is not a square (True)

25. a △ with sides 8, 9 & 10 is not right angled.

If right angled then use pythag.

$10^2 = 8^2 + 9^2$

$100 = 64 + 81$

$100 \neq 81$

26. $6p + 10q = 157$

$2[3p + 5q] = 157$

157 cannot be ÷ by 2 evenly

∴ there exists no $p \in q$

27. $\frac{a}{b} + \frac{b}{a} \geq 2$ if $a = b$

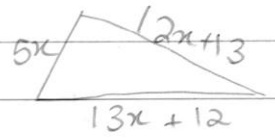
$\frac{a}{a} + \frac{b}{b} \geq 2$ $1 + 1 \geq 2$ true

if $a > b$ $\frac{a}{b} + \frac{b}{a} \geq 2$

↑ Improper fraction

Misc Ex One.

28.



$a^2 + b^2 = c^2$

$(5x)^2 + (12x+13)^2$

$25x^2 + 144x^2 + 312x + 169$

$= 169x^2 + 312x + 169$

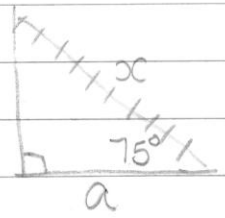
$(13x + 12)^2$

$= 169x^2 + 312x + 144$

$169x^2 + 312x + 169 \neq 169x^2 + 312x + 144$

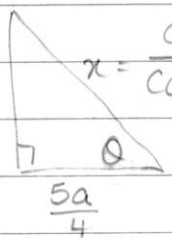
$\therefore$ not right angled $\triangle$

1.



$\cos 75^\circ = \frac{a}{x}$

$x = \frac{a}{\cos 75}$



$\cos\theta = \frac{\frac{5a}{4}}{x}$

$\cos^{-1}\left(\frac{\frac{5a}{4}}{\frac{a}{\cos 75}}\right)$

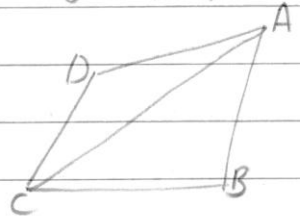
$\cos^{-1}\left(\frac{5a}{4} \times \frac{\cos 75}{a}\right)$

$\cos^{-1}\left(\frac{5}{4}\cos 75\right)$

$\cos^{-1}(0.3235)$

$\theta = 71.1^\circ$

2.

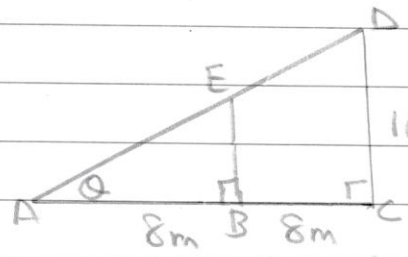


In $\triangle ADC$ angle sum = 180

In $\triangle ABC$ angle sum = 180

$\therefore$ angle sum of quadrilateral = 360°

3.



In $\triangle AEB$ & In $\triangle ADC$

$\angle EAB = \angle DAC$ (common angle)

$\angle EBA = \angle DCA$ (90° given)

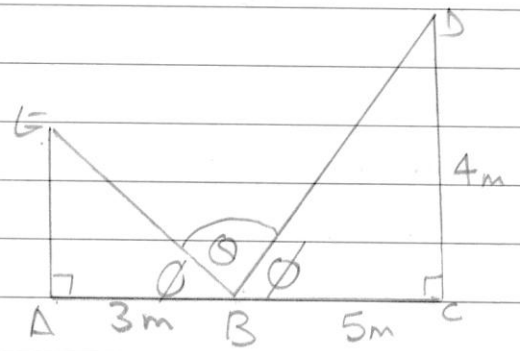
$\therefore \angle AEB = \angle ADC$ (sum of $\triangle = 180^\circ$)

$\therefore \triangle AEB \sim \triangle ADC$ (AAA)

$\therefore \frac{EB}{8} = \frac{10}{16}$ $EB = \frac{10}{16} \times 8$

$\therefore EB = 5cm$

4.



given $\angle EBD = 180 - 2 \times \angle EBA$

$\therefore \angle EBA = \angle DBC$

In $\triangle EAB$ & In $\triangle DCB$

$\angle EAB = \angle DCB$ (90° given)

$\angle EBA = \angle DBC$ (from above)

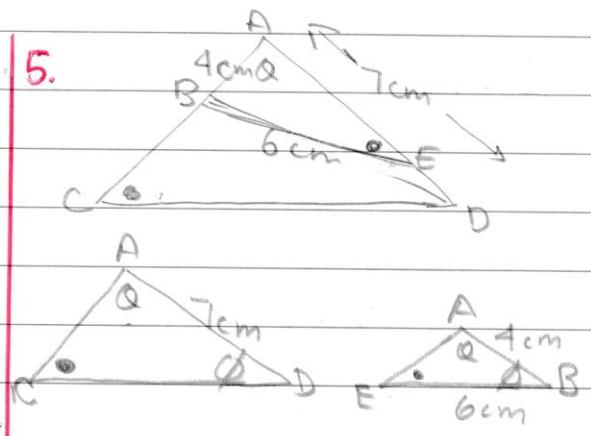
$\therefore \angle AEB = \angle CDB$ ($\triangle$ sum $= 180^\circ$)

$\therefore \triangle EAB \sim \triangle DCB$ (AAA)

$\therefore \frac{EA}{3} = \frac{4}{5}$ $\therefore EA = 2.4m$

$EA = \frac{4}{5} \times 3$

5. 5.



In $\triangle ACD$ & In $\triangle AEB$

$\angle ACD = \angle AEB$ (given)

$\angle CAD = \angle EAB$ (common $\angle$)

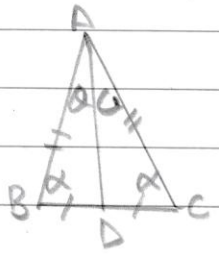
$\therefore \angle ADC = \angle ABE$ ($\triangle$ Sum $= 180^\circ$)

$\therefore \triangle ACD \sim \triangle AEB$ (AAA)

$\therefore \frac{CD}{7} = \frac{6}{4}$

$\therefore CD = \frac{6}{4} \times 7 = 10.5$

6.



$\triangle ABC$ isosceles (given)

$\therefore AB = AC$ (given)

AD is drawn such that

$BD = DC$ (given)

$\angle ABD = \angle ACD$ (given)

$AD = AD$ (common side)

$\therefore \triangle ABD \equiv \triangle ACD$ (S.S.S)

let $\angle BAD = \theta$ $\therefore \angle DAC = \theta$

let $\angle ABD = \alpha$ $\therefore \angle ACD = \alpha$

$\triangle ABC$ angle sum is 180°

$\therefore 2\theta + 2\alpha = 180$

$2(\theta + \alpha) = 180$

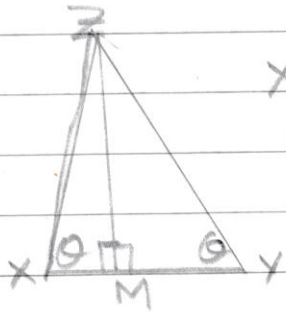
$\theta + \alpha = 90$

$\therefore$ in $\triangle ABD$ $\angle ADB = 90^\circ$

$\therefore$ in $\triangle ACD$ $\angle ADC = 90^\circ$

$\therefore$ line AD is $\perp$ to line BC

7.



$XZ = YZ$

$\triangle XYZ$ is isosceles (given)

$ZM \perp XY$ (given)

$\triangle ZXY$ isosceles $\therefore \angle ZXY = \angle ZYX$

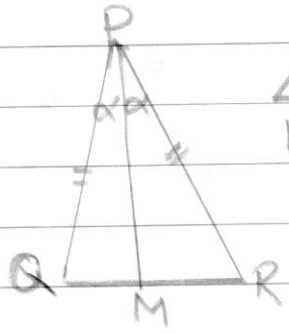
$ZM = ZM$

$\therefore \triangle ZMX \equiv \triangle ZMY$ (RHS)

if $XM = MY$

$\therefore$ M is midpoint of XY

8.



$\angle QPR$ is bisected (given)

$\angle QPM = \angle RPM$ (given)

$QP = RP$ (given)

$PM = PM$ (common side)

$\therefore \triangle PQM \equiv \triangle PRM$ (S.A.S)

$\therefore QM = MR$

$\therefore$ M is midpoint of QR